Løsningsforslag

Matematik A 17. januar 2006

Opgave 1

a)

$$A_{M_1} = -\int_{\pi/2}^{3/2\pi} \cos x dx = -\left[\sin x\right]_{\pi/2}^{3/2\pi} = -(-1-1) = \underline{2}$$

b)

$$V_{x,M_1} = \pi \int_{0}^{\frac{\pi}{2}} (\cos x)^2 dx = \pi\left[\frac{1}{2}x + \frac{1}{4}\sin 2x\right]_{0}^{\frac{\pi}{2}} = \cancel{\pi\left(\frac{1}{4}\pi - 0 - \frac{\pi}{4} - 0\right) = \frac{\pi^2}{2} \approx 4,93}$$

$$= \pi\left(\frac{1}{2}\cdot\frac{\pi}{2} + \frac{1}{4}\cdot\sin 2\cdot\frac{\pi}{2} - 0 - \frac{1}{4}\sin 0\right)$$

$$= \pi\left(\frac{\pi}{4} + 0 - 0 - 0\right) = \frac{\pi^2}{4} = \underline{\underline{2,47}}$$

c)

Øvre grænse : $\cos x = \frac{1}{2} \wedge x \in \left[0; \frac{\pi}{2}\right] \Rightarrow x = \frac{\pi}{3}$

$$A_{M_3} = \int_0^{\pi/3}\left(\cos x - \frac{1}{2}\right)dx = \left[\sin x - \frac{1}{2}x\right]_0^{\pi/3} = \frac{\sqrt{3}}{2} - \frac{\pi}{6} - 0 + 0 = \underline{\frac{\sqrt{3}}{2} - \frac{\pi}{6}} \approx 0,342$$

d)

$$V_{y,M_3} = 2\pi\int_0^{\pi/3} x\left(\cos x - \frac{1}{2}\right)dx = 2\pi\int_0^{\pi/3}\left(x\cos x - \frac{1}{2}x\right)dx = 2\pi\left[x\sin x + \cos x - \frac{1}{4}x^2\right]_0^{\pi/3} =$$

$$2\pi\left(\frac{\pi}{3}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2} - \frac{1}{4}\cdot\frac{\pi^2}{9} - 0 - 1 + 0\right) = 2\pi\left(\frac{\pi\sqrt{3}}{6} - \frac{1}{2} - \frac{\pi^2}{36}\right) = \frac{\pi\left(6\pi\sqrt{3} - 18 - \pi^2\right)}{18} \approx$$

<u>0,834</u>

Opgave 2

a)

$$\frac{dy}{dx} = x^3 y^2 \Leftrightarrow$$

$$\int \frac{1}{y^2} dy = \int x^3 dx + k_1 \quad , \quad k_1 \in R \quad , \quad y \neq 0 \Leftrightarrow$$

$$-\frac{1}{y} = \frac{1}{4} x^4 + k_1 \Leftrightarrow$$

$$\frac{1}{y} = -\frac{1}{4} x^4 - k_1 \Leftrightarrow$$

$$y = \frac{1}{\frac{1}{4} x^4 - k_1} \Leftrightarrow$$

$$\underline{y = \frac{4}{x^4 + k_2} \quad , \quad k_2 \in R \quad , \quad x^4 + k_2 \neq 0} \quad ; \quad \underline{y = 0} \text{ er også løsning (ses ved indsættelse)}$$

b)

$$f(1) = -2 \Leftrightarrow \frac{4}{1 + k_2} = -2 \Leftrightarrow 1 + k_2 = -2 \Leftrightarrow k_2 = -3 \Rightarrow$$

$$\underline{f(x) = \frac{4}{x^4 - 3} \quad , \quad x^4 - 3 \neq 0}$$

$$g(-1) = 0 \Rightarrow \underline{y = 0}$$

Opgave 3

a)

$x=0 \Leftrightarrow t^2-2=0 \Leftrightarrow t^2=2 \Leftrightarrow t=\sqrt{2} \vee t=-\sqrt{2} \Rightarrow$

$\underline{(x,y)=(0,-6-2\sqrt{2}) \approx (0;-8,83),(x,y)=(0,-6+2\sqrt{2}) \approx (0;-3,17)}$

$y=0 \Leftrightarrow t^2-2t-8=0 \Leftrightarrow t=-2 \Leftrightarrow t=4 \Rightarrow \underline{(x,y)=(14,0),(x,y)=(2,0)}$

b)

$x'=0 \Leftrightarrow 2t=0 \Leftrightarrow t=0$

$y'=0 \Leftrightarrow 2t-2=0 \Leftrightarrow 2t=2 \Leftrightarrow t=1$

Lodret tangent : $t=0 \Rightarrow \underline{(x,y)=(-2,-8)}$

Vandret tangent : $t=1 \Rightarrow \underline{(x,y)=(-1,-9)}$

c)

$\overrightarrow{OP}=\begin{pmatrix} x(1) \\ y(1) \end{pmatrix}=\begin{pmatrix} -1 \\ -9 \end{pmatrix}$, dvs. et punkt på tg. er P $(-1,-9)$

$\overrightarrow{v(1)}=\begin{pmatrix} x'(1) \\ y'(1) \end{pmatrix}=\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, dvs. tangenten er vandret (se også spm. b)) og $\underline{y=-9}$

d)

$\overrightarrow{r'(t)} \parallel \vec{v} \Leftrightarrow \overrightarrow{r'(t)} \cdot \vec{v}=0 \Leftrightarrow \begin{pmatrix} 2t \\ 2t-2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \end{pmatrix}=0 \Leftrightarrow -6t+4t-4=0 \Leftrightarrow -4=2t \Leftrightarrow t=-2$

$\overrightarrow{OP}=\begin{pmatrix} x(-2) \\ y(-2) \end{pmatrix}=\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, dvs. et punkt på tg. er P $(-1,-9)$

$\overrightarrow{v(1)}=\begin{pmatrix} x'(-2) \\ y'(-2) \end{pmatrix}=\begin{pmatrix} -4 \\ -6 \end{pmatrix}=-2\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\underline{\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 2 \\ 0 \end{pmatrix}+s\begin{pmatrix} 2 \\ 3 \end{pmatrix}, s \in R}$

e)

$y=-9 \Rightarrow 3s=-9 \Leftrightarrow s=-3 \Rightarrow x=2-3\cdot 2=-4$, dvs. $\underline{S(-4,-9)}$

$$\cos\angle(t_1,t_2)=\frac{\begin{pmatrix}2\\0\end{pmatrix}\cdot\begin{pmatrix}-4\\-6\end{pmatrix}}{\sqrt{2^2+0^2}\sqrt{(-4)^2+(-6)^2}}=\frac{2\cdot(-4)+0\cdot(-6)}{2\sqrt{52}}=\frac{-8}{4\sqrt{13}}=-\frac{2}{\sqrt{13}}\approx -0,5547\Rightarrow$$

$\underline{\angle(t_1,t_2)\approx 123,69°}$ (eller 56,31°)

Opgave 4

a)

$$2(x-0)-5(y-2)+3(z-12)=0 \Leftrightarrow 2x-5y+10+3z-36=0 \Leftrightarrow$$
$$\underline{2x-5y+3z-26=0}$$

b)

$$l\ :\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \overrightarrow{OB}+t\overrightarrow{BC} = \begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4-5 \\ -2-(-2) \\ 3-0 \end{pmatrix}\ ,\ t \in R \Leftrightarrow$$
$$\underline{l\ :\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix} + t\begin{pmatrix} -9 \\ 0 \\ 3 \end{pmatrix}\ ,\ t \in R}$$

c)

$$\cos\angle\left(\vec{n}_\alpha,\vec{r}_l\right) = \frac{\vec{n}_\alpha \cdot \vec{r}_l}{\left|\vec{n}_\alpha\right|\left|\vec{r}_l\right|} = \frac{2\cdot(-9)-5\cdot 0+3\cdot 3}{\sqrt{2^2+(-5)^2+3^2}\sqrt{(-9)^2+0^2+3^2}} = \frac{-9}{6\sqrt{19}\sqrt{5}} \approx -0{,}1539 \Rightarrow$$

$$\angle\left(\vec{n}_\alpha,\vec{r}_l\right) \approx 98{,}85° \Rightarrow \underline{\angle(\alpha,l) = 90° - \angle\left(\vec{n}_\alpha,\vec{r}_l\right) \approx 8{,}85°}$$

$$2x-5y+3z-26=0 \wedge x=5-9t \wedge y=-2 \wedge z=3t \Rightarrow$$
$$10-18t+10+9t-26=0 \Leftrightarrow -9t=6 \Leftrightarrow t=-\frac{2}{3} \Rightarrow S\left(5-9\cdot\left(-\tfrac{2}{3}\right),-2,3\cdot\left(-\tfrac{2}{3}\right)\right)=$$
$$\underline{S(11,-2,-2)}$$

d)

$$5-9t=3+s \wedge -2=2+4s \wedge 3t=7+6s \Leftrightarrow 4s=-4 \wedge t=\tfrac{7}{3}+2s \wedge s=2-9t \Leftrightarrow$$
$$s=-1 \wedge t=\tfrac{7}{3}-2=\tfrac{1}{3} \wedge -1=2-3 \quad \text{(kontrol)}\text{, dvs.}$$

$$S(3-1,2-4,7-6)=\underline{S(2,-2,1)}$$

e)

α og m er parallelle, idet $\overrightarrow{n_\alpha} \cdot \overrightarrow{r_m} = 0$ $(2 \cdot 1 - 5 \cdot 4 + 3 \cdot 6 = 2 - 20 + 18 = 0)$, dvs. afstanden fra α til m er den samme som afstanden mellem et punkt på m (D) og α :

$$dist(m,\alpha) = \frac{|2 \cdot 3 - 5 \cdot 2 + 3 \cdot 7 - 26|}{\sqrt{2^2 + (-5)^2 + 3^2}} = \frac{|-9|}{\sqrt{38}} = \frac{9}{\sqrt{38}} \approx 1{,}46$$

Opgave 5

a)

$$\int \frac{e^{2x}-3e^{x}}{e^{x}}dx = \int \left(e^{x}-3\right)dx = \underline{e^{x}-3x+k \ , \ k \in R}$$

b)

$$\int_{0}^{1}\left(2x^{2}+1\right)\ln\left(2x^{3}+3x+1\right)dx = \frac{1}{3}\int_{1}^{6}\ln t dt = \frac{1}{3}\left[t\ln t - t\right]_{1}^{6} = \frac{1}{3}\left(6\ln 6-6-0+1\right) =$$

$$\underline{\frac{1}{3}\left(6\ln 6-5\right)} \approx 1{,}92$$

idet $t = 2x^{3}+3x+1 \Rightarrow \frac{dt}{dx} = 6x^{2}+3 = 3\left(2x^{2}+1\right) \Leftrightarrow \frac{1}{3}dt = \left(2x^{2}+1\right)dx$.

c)

$$\int \left(2x^{2}+1\right)\ln x dx = \left(\frac{2}{3}x^{3}+x\right)\ln x - \int \frac{1}{x}\left(\frac{2}{3}x^{3}+x\right)dx = \left(\frac{2}{3}x^{3}+x\right)\ln x - \int\left(\frac{2}{3}x^{2}+1\right)dx =$$

$$\underline{\left(\frac{2}{3}x^{3}+x\right)\ln x - \frac{2}{9}x^{3}-x+k \ , \ k \in R}$$

idet $\left(\ln x\right)' = \frac{1}{x}$ og $\int\left(2x^{2}+1\right)dx = \frac{2}{3}x^{3}+x$.